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Identification of IIR Systems Using Harmony Search Algorithm

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Abstract—**Adaptive infinite impulse response (IIR) systems are widely used in modeling real world problems. This ubiquitous class of models requires less parameters and evince better performance over finite impulse response (FIR) systems. In the present paper, the system identification problem of IIR models is translated into a nonlinear optimization problem and a recently introduced population based algorithm, harmony search (HS), is adopted to cope with the identification problem. Furthermore, the performance of the proposed methodology is compared with two well-known meta-heuristic algorithms, genetic algorithm (GA) and particle swarm optimization (PSO). The identification results pertaining to two benchmark IIR systems are included which demonstrate that the proposed method is superior to GA and PSO based algorithms in terms of convergence speed and estimation accuracy.**

I. Introduction

Adaptive IIR system identification has received considerable attention in open literature [1]-[12]. Generally speaking, IIR systems are more auspicious than FIR models. This stems from the fact that an IIR structure require less number of parameters to model a real world system with a given performance criterion; thus, it entails less computational burden compared to a FIR structure. This virtue is due to the fact that IIR systems encompass both poles and zeros; whereas, FIR systems just allow for zeros [1].

Classical derivative based identification methods for IIR systems have been addressed by a number of researchers [1]-[3]. In [3], Johnson brought forward a tutorial in which the concept of adaptive control and adaptive filtering are correlated. In the survey paper [1], Shynk presented an expository overview of the classic methods. However, the performance of conventional derivative based methods when applied to adaptive IIR filtering problems is marred by several glitches. First of all, the error surface associated with adaptive IIR filtering problems are multi-modal and non-quadratic with respect to filter coefficients. Therefore, gradient based algorithms (e.g. least mean square), which advance toward the direction of negative gradient, get trapped in local minima hence failing to converge to global minima [13]. Another important issue is that the bounded input bounded output (BIBO) stability of the filter should be supervised during the identification process, since the corresponding filter poles can readily escape from the unit circle.

One avenue that researchers have followed in their attempt to surmount the deficiencies of derivative based identification algorithms is the implementation of evolutionary and swarm intelligence algorithms [4]-[12]. In a seminal paper [4], Ng *et al.* proposed a hybrid algorithm incorporating GA and LMS method for adaptive IIR filtering. In a series of subsequent papers [5]-[7], Abe *et al.* introduced the notion of evolutionary digital filtering. Following the same trend, several other investigators utilized meta-heuristic algorithms for IIR system identification e.g. bee colony optimization [8], PSO with quantum infusion [9], PSO and bacterial foraging optimization [10], seeker optimization [11], cat swarm optimization [12], and etc.

In this paper, a novel IIR identification algorithm based on HS method is introduced. The HS algorithm first brought forward in [14] imitates the harmony search process performed by musicians. Similar to previously suggested meta-heuristic algorithms, the HS algorithm is derivative free and it requires fewer mathematical operations. The performance of the HS based adaptive IIR filtering strategy is verified by numerous simulation analyses. Specifically, it is shown that the convergence time of HS is considerably lower than conventional algorithms which make it appropriate for real-time applications.

The rest of this paper is organized as follows. The subsequent section considers a brief introduction to IIR system identification. Section III discusses the HS algorithm. The proposed HS based IIR system identification method is delineated in section IV. Simulation results and some discussions are given in section V. The paper ends with conclusions in section VI.

II. IIR System Identification

The problem of determining a mathematical model for an unknown system by monitoring its input-output data is known as system identification [15]-[17]. The task of any given parametric system identification algorithm is to vary the model parameters until a pre-defined approximation criterion is satisfied. The block diagram of an arbitrary IIR system identification algorithm is shown in Fig. 1. The adaptive algorithm essays to tune the adaptive filter coefficients such that the error between the output of the unknown system and the estimated output is minimized. An IIR system is described as:

$$Y(z) = H(z)\, U(z) \tag{1}$$

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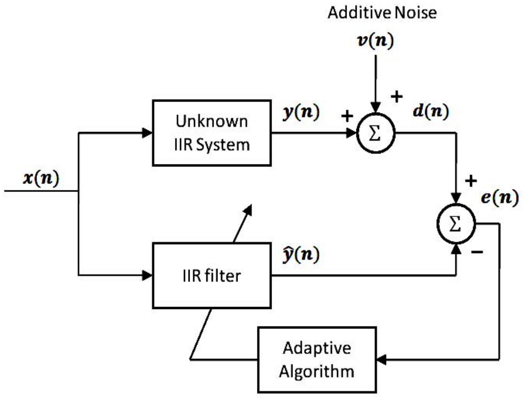

Fig. 1. Block diagram of an IIR system identification algorithm

wherein,

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m}}{1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}} \quad (2)$$

is the IIR transfer function, $Y(z)$ is the z-transform of the output $y(n)$, and $U(z)$ denotes the z-transform of the input $u(n)$. a_i, $i = 0,1,2,\ldots,m$ and b_i, $i = 1,2,3,\ldots,n$ are the feed-forward and feed-back coefficient of the IIR system, respectively. The IIR filter can be formulated as a difference equation

$$y(n) = -\sum_{k=1}^{n} b_k\, y(n-k) + \sum_{k=0}^{m} a_k\, u(n-k) \quad (3)$$

As illustrated in Fig. 1, $v(n)$ is the additive noise in the output of the system. Combining $v(n)$ and $y(n)$, we get the overall output of the system $d(n)$. Additionally, for the same set of inputs, the adaptive IIR filter block engenders $\hat{y}(n)$. The estimated transfer function can be represented as

$$\hat{H}(z) = \frac{\hat{a}_0 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2} + \cdots + \hat{a}_m z^{-m}}{1 + \hat{b}_1 z^{-1} + \hat{b}_2 z^{-2} + \cdots + \hat{b}_n z^{-n}} \quad (4)$$

where $\hat{a}_i$ and $\hat{b}_i$ signify the approximated coefficients of the IIR system. In other words, the transfer function of the actual system is to be identified using the transfer function of the adaptive filter. The difference between $d(n)$ and $\hat{y}(n)$ produces the input to the adaptive algorithm .The adaptive algorithm uses this residual to adjust the parameters of the IIR system. It can be discerned from the figure that

$$d(n) = y(n) + v(n) \quad (5)$$

$$e(n) = d(n) - \hat{y}(n) \quad (6)$$

The cost function (mean square error) to be minimized by the adaptive identification algorithm is given by

$$J = E\left[(d(n) - \hat{y}(n))^2\right] \cong \frac{1}{N}\sum_{n=1}^{N} e^2(n) \quad (7)$$

where, N denotes the number of input samples and $E(.)$ is the statistical expectation operator. The optimization algorithms employed in this paper search the solution space to locate those values of parameters which contribute to the minimization of Eq. (7).

III. Harmony Search Algorithm

HS algorithm is a relatively new music inspired meta-heuristic optimization algorithm. Since its emergence in [14], it has been applied to a surfeit of real world optimization problems [18]-[21]. The ultimate goal of music is to reach a consummate state of harmony. In this case, the musician's improvisation process can be compared to an optimization process seeking the finest attainable harmony. The harmonics which a specific music instrument can produce is chiefly reliant on the pitch or frequency range pertaining to that instrument.

Three notions are mainly used throughout the HS algorithm [22]-[24]: the use of harmony memory, pitch adjusting, and randomization. The use of harmony memory is vital because it corresponds to selecting the fittest individuals in GA. This would guarantee that the preeminent harmonies will be conducted to the new memory. For the sake of using this memory further efficaciously, a parameter $r_{accept} \in [0,1]$, called harmony memory acceptance rate (HMAR), is introduced. For a lower HMAR, less number of best harmonies will be chosen, resulting in a slower convergence speed. For extremely high values of HMAR (close to 1), nearly all the harmonies are exploited in the harmony memory. As a consequence, the remaining harmonies are not examined properly, precipitating in possibly a pre-mature convergence. So, we take HMAR to be $r_{accept} \in [0.7, 0.95]$.

The adjustment of the pitch can be carried out linearly or nonlinearly; but, linear adjustment is preferable for empirical reasons. Let p_{old} be the current solution (or pitch), then the new solution (pitch) p_{new} is calculated as

$$p_{new} = p_{old} + b_p\,(2 * rand - 1) \quad (8)$$

where $rand$ is a random number drawn from a uniform distribution in the span of $[0,1]$. b_p stands for the bandwidth given by

$$b_p = \frac{p_{range}}{W} \quad (9)$$

in which, W is the adjust rate for pitch adjusting, p_u is upper pitch limit (upper bound of the search space), p_l is the lower pitch limit (lower bound of the search space), and $p_{range} = p_u - p_l$ is the pitch range. The bandwidth restrains the local range of pitch adjustment. Indeed, the pitch adjustment is performed randomly. In addition, a pitch-adjusting rate (r_{pa}) is assigned which determines the level of adjustment. For lower values of r_{pa}, pitch values experience trivial alterations. For extremely high values, the algorithm may display divergent behavior. An appropriate range for r_{pa} is $[0.1, 0.5]$.

The last component is the randomization, which ensures the diversity of candidate solutions. It is worth noting that the randomness of the pitch adjustment stage is limited to a local search, and therefore is not enough to make sure that the search space is well scrutinized. The implementation of a randomization step can oblige the algorithm further to inspect various regions with high solution diversity in order to achieve global optimality. Consequently, we have

$$p_{new} = p_l + p_{range} * rand \quad (10)$$

The flow of the HS algorithm can be summarized in the pseudo-code given in Fig. 2.

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Harmony Search Algorithm

1.Generate initial harmonics (solution vectors).
  a. Define pitch adjusting rate, harmony memory acceptance rate, Max_iter and
     pitch limits.
2.while (iter < Max_iter)
  a. Accept the best harmonics in the solution vector (the harmonics which
     corresponds to the minimum objective function value).
  b. Adjust pitch to get new harmonics (solutions):
     if (rand > harmony memory acceptance rate)
        Randomly, choose an existing harmonic (memory).
     else if (rand > pitch adjusting rate)
        Adjust the pitch randomly within a bandwidth according to equation (8).
     else
        Generate new harmonics according to equation (10). (New search via
        randomization)
     end if
  c. Evaluate the objective function for the new solution.
  d. Accept the new solution if it have a better fitness.
 end while
3.Find the current best estimates
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Fig. 2. The pseudo-code of the HS algorithm.

From the above discussions, it can be deduced that HS requires less mathematical computations with respect to GA and PSO. Also, there is evidence to suggest that HS is less sensitive to the chosen parameters, which means that one does not need to fine-tune these parameters to get high quality solutions [24].

IV. HS Based IIR System Identification

In order to integrate the advantages of HS optimization method in IIR adaptive filtering, the following identification algorithm is proposed:

1) Initially, for a set of given input-output pairs $\{x(i), y(i)\}_{i=1}^{N}$, preset HS parameters r_{accept}, r_{pa}, Max_iter and W. Construct a matrix H of size $p \times q$, where p represents the population size or the number of the harmonies and q refers to the population dimension or the number of pitches that produces the harmony. In case of IIR system identification, q corresponds to the number of adaptive IIR model's coefficients. Each component of H is initialized randomly in the search space.

2) For $k = 1,2, \dots, p$, compute a set of outputs, pertaining to k'th harmony, from the adaptive model $\hat{y}_k(i)$, $i = 1,2, \dots, N$. Subsequently, evaluate the fitness associated with the k'th harmony according to the following equation

$$J_k = \frac{1}{N}\left[\left(Y - \hat{Y}_k\right)^T \left(Y - \hat{Y}_k\right)\right] \tag{11}$$

where, $Y = [y(1)\ y(2) \dots y(N)]^T$, and $\hat{Y}_k = [\hat{y}_k(1)\ \hat{y}_k(2) \dots \hat{y}_k(N)]^T$. Then, create a vector $\mathcal{J} = [J_1\ J_2 \dots J_p]^T$. Set $iter = 1$.

3) If $iter < Max\_iter$, for $j = 1,2, \dots, q$, perform steps 2.a and 2.b of the HS algorithm (see Fig. 2.) and store the new solution.

3. a. Evaluate the fitness function for each new solution $\hat{J}$.

3. b. Find the maximum value in $\mathcal{J}$, J_{max} and use its index l to find the corresponding row in H.

3. c. If $\hat{J} < J_{max}$, move the new solution along with its fitness $\hat{J}$ to the l'th row of H and $\mathcal{J}$, respectively. Set $iter = iter + 1$.

3. d. Otherwise, check the next solution. Set $iter = iter + 1$.

4) Save the harmony (filter coefficients) which corresponds to the least fitness (best attainable match between the IIR model and the actual system in the sense of MSE).

V. Results and Discussions

In this section, we will consider two test scenarios. In both cases, two IIR models are used: one with the same order as the actual system, and another with a reduced order IIR structure. As the number of coefficients decreases the degree of freedom reduces and it becomes more difficult to identify the actual system. In order to ensure the validity of the results, each experiment is repeated in 20 consecutive trials and the resultant (best, standard deviation, and mean) values are given in corresponding tables. For the sake of a more comprehensive comparison, in addition to HS, simulations provided in this section are done using standard versions of GA and PSO, as well. Each simulation is carried out in MATLAB v.7.5 on the same computer with Intel® Core™ 2 Duo CPU T7250 @ 2.00 GHz and 1.00 GB of RAM. In all cases the population size is set to 50, and the input data is a Gaussian white noise with zero mean and unit variance. The output data is contaminated with a Gaussian random noise with zero mean and a variance of 0.001. Moreover, the HS parameters are set as $r_{pa} = 0.5$, $r_{accept} = 0.95$, and $W = 100$. Also, in the GA algorithm the bit number is set to 16, mutation probability is 0.1, and crossover step is of single point type with a probability of 0.7. The parameters of PSO are given next. Acceleration constants are set to 2, inertia weight is set to 0.5, and the random numbers take values between 0 and 1.

Example 1. Consider the following IIR system

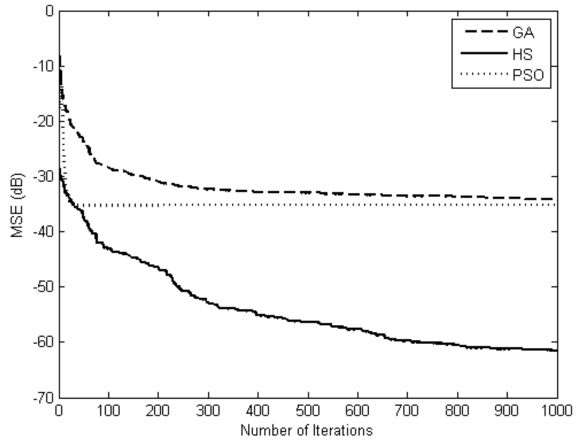
$$H(z) = \frac{0.05 - 0.4z^{-1}}{1 - 1.1314z^{-1} + 0.25z^{-2}} \tag{12}$$

System (12) is identified using the following IIR structures

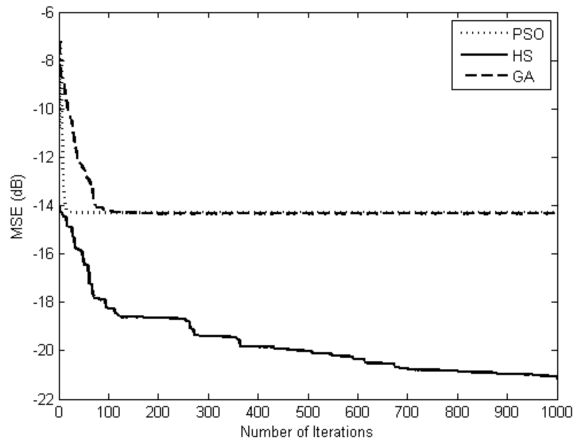
$$H(z) = \frac{a_0 + a_1 z^{-1}}{1 - b_1 z^{-1} - b_2 z^{-2}} \tag{13}$$

$$H(z) = \frac{a_0}{1 - b_1 z^{-1}} \tag{14}$$

The simulation results related to the actual order model (13) and reduced order model (14) are given in Figs. 3 and 4, respectively. In all figures the vertical axis denotes the magnitude in dB and the horizontal axis stands for the number of iterations. It is clear from the figures that the utilization of HS has resulted in greater estimation



(a)



(b)

Fig. 3. MSE performance of the GA, PSO, and HS based IIR system identification algorithms: actual order (a) reduced order (b).

accuracy. Table I and Table II give the attained MSE values over 20 simulations regarding the actual order and reduced order models, respectively. As it is observed, in all tables the best values are given in bold. The respective computation time of the algorithms related to the actual and reduced model are listed in Table III and IV, as well. Besides, Table V illustrates the mean values of the estimated filter coefficients. The results demonstrate that HS based adaptive IIR filtering algorithm is much faster than the GA and PSO. It also brings about superior model matching performance.

Example 2. Consider the following IIR system

$$H(z) = \frac{-0.2-0.4z^{-1}+0.5z^{-2}}{1-0.6z^{-1}+0.25z^{-2}-0.2z^{-3}} \quad (15)$$

The two IIR structures used for the identification purpose are given below

$$H(z) = \frac{a_0+a_1z^{-1}+a_2z^{-2}}{1-b_1z^{-1}-b_2z^{-2}-b_3z^{-3}} \quad (16)$$

$$H(z) = \frac{a_0+a_1z^{-1}}{1-b_1z^{-1}-b_2z^{-2}} \quad (17)$$

TABLE I
ACHIEVED MSE VALUES FOR EXAMPLE 1 (ACTUAL ORDER)

Example 1-actual	HS	PSO	GA
Best	8.3730e-4	0.0173	0.0196
Average	**0.0033**	0.0188	0.0292
Std.dev.	0.0047	0.0162	0.0265

TABLE II
ACHIEVED MSE VALUES FOR EXAMPLE 1 (REDUCED ORDER)

Example 1-reduced	HS	PSO	GA
Best	0.0878	0.1927	0.1916
Average	**0.1053**	0.1939	0.1980
Std. dev.	0.0203	0.0140	0.0265

TABLE III
THE CONVERGENCE SPEED OF THE IDENTIFICATION METHODS IN SECONDS (ACTUAL ORDER)

Computational Time	HS	PSO	GA
Average	**0.0709**	6.3173	348.3820
Best	0.0683	6.1786	343.3637
Worst	0.0743	6.9776	355.5382
Std. dev.	0.0017	0.2207	3.1313

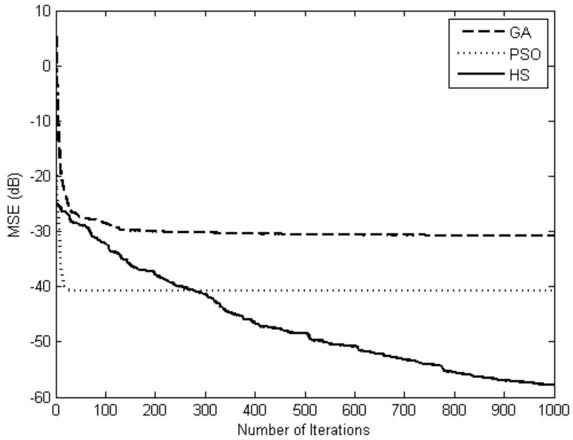
TABLE IV
THE CONVERGENCE SPEED OF THE IDENTIFICATION METHODS IN SECONDS (REDUCED ORDER)

Computational Time	HS	PSO	GA
Average	**0.0543**	5.1368	360.8027
Best	0.05186	4.9301	344.3717
Worst	0.0566	5.8694	378.9104
Std. dev.	0.0014	0.2184	10.8203

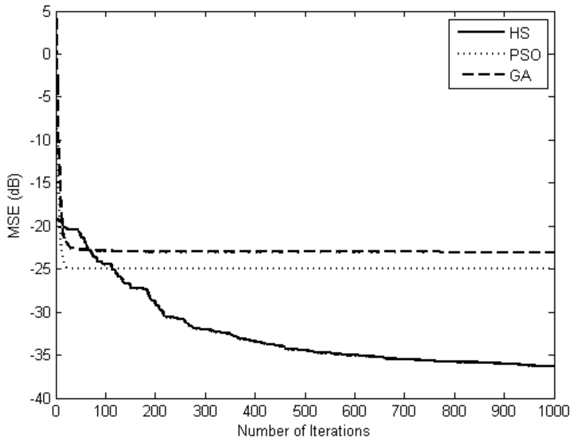
TABLE V
THE MEAN VALUES OF ESTIMATED PARAMETERS

Coefficient	Actual Value	HS	PSO	GA
a_0	0.05	**0.0506**	-0.0032	0.0362
a_1	-0.4	**-0.4019**	-0.3752	-0.4491
b_1	1.1314	**1.1817**	0.9838	0.9094
b_2	-0.25	**-0.2599**	-0.1088	-0.0440

Model (16) has the same order as system (15); but, the order of (17) is lower than (15). The same set of simulations as executed in example 1 has been launched. Fig. 4 represents the average MSE graphs from 20 simulation tests. It is discerned from the figures that unlike GA and PSO, which after a number of iteration cycles their MSE values become steady, the MSE values pertaining to HS continue to decrease as the iteration numbers augment. Thus, HS has a better chance of averting local minima. Moreover, it should be noted that HS corresponds to the least MSE values. This can be further corroborated by inspecting Table VI and VII which include the respective MSE values for identifying models (16) and (17).



(a)



(b)

Fig. 4 MSE performance of the GA, PSO, and HS based IIR system identification algorithms: actual order (a) reduced order (b).

The computational time of an adaptive filtering algorithm is a critical issue in real-time applications. Table VIII and Table IX give the convergence speeds of the three algorithms under study in 20 simulation experiments. Again, HS based identification methodology has the best r. The mean values of the approximated coefficients of the IIR filter are also depicted in Table X.

All in all, the extensive simulation results given in this section establishes that HS is a vigorous optimization tool in IIR adaptive filtering and IIR system identification problems.

VI. Conclusion

In this paper, the identification problem of IIR systems based on HS algorithm was considered. By converting the system identification task into a complex optimization problem, the HS was readily utilized. Extensive simulation results were reported which ascertained the validity of the proposed HS based adaptive IIR filtering algorithm. It was also demonstrated through simulation analysis that in contrast to conventional meta-heuristic algorithms, like GA and PSO, HS has a minor chance of pre-mature convergence. In addition, it has been shown that HS based IIR system identification method would result in a much less computational complexity. Therefore, the proposed method can be employed in real-time tasks, for instance, in adaptive signal processing applications. A suggested subject for future research could be to combine HS with other algorithms like PSO to improve the convergence speed.

TABLE VI
Achieved MSE Values for Example 2 (Actual Order)

Example 2-actual	HS	PSO	GA
Best	0.0013	0.0092	0.0288
Average	**0.0085**	0.0095	0.0361
Std.dev.	0.0110	0.0043	0.0720

TABLE VII
Achieved MSE Values for Example 2 (Reduced Order)

Example 2-reduced	HS	PSO	GA
Best	0.0161	0.0564	0.0704
Average	**0.0331**	0.0578	0.0753
Std.dev.	0.0276	0.0176	0.0639

TABLE VIII
The Convergence speed of The Identification Methods in Seconds (Actual Order)

Computational Time	HS	PSO	GA
Average	**0.0886**	7.6719	361.5028
Best	0.0869	7.3638	343.2597
Worst	0.0911	8.0980	393.4641
Std.dev	0.0013	0.2112	13.7960

TABLE IX
The Convergence speed of The Identification Methods in Seconds (Reduced Order)

Computational Time	HS	PSO	GA
Average	**0.0749**	6.2917	379.0929
Best	0.0703	6.1290	365.5978
Worst	0.0802	6.4578	397.5246
Std.dev	0.0035	0.0854	10.1545

TABLE X
The Mean Values of Estimated Parameters

Coefficient	Actual Value	HS	PSO	GA
a_0	-0.2	**-0.2065**	-0.2177	-0.2069
a_1	-0.4	-0.4219	**-0.3997**	-0.5470
a_2	0.5	**0.4561**	0.3411	0.0901
b_1	0.6	**0.4707**	0.3766	-0.1036
b_2	-0.25	-0.3188	**-0.2637**	-0.3570
b_3	0.2	**0.2288**	0.1196	-0.0836

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